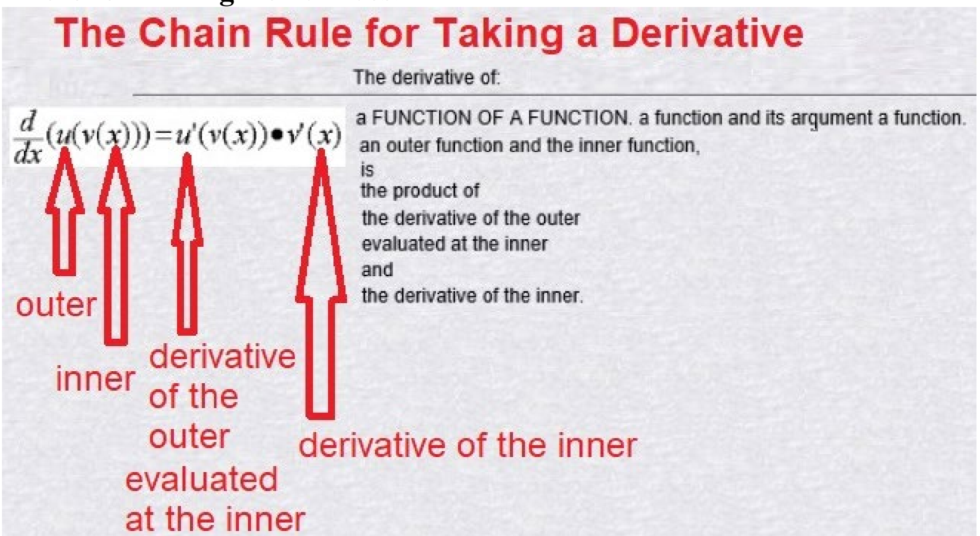
Take an Antiderivative by Substitution   pdf of this page

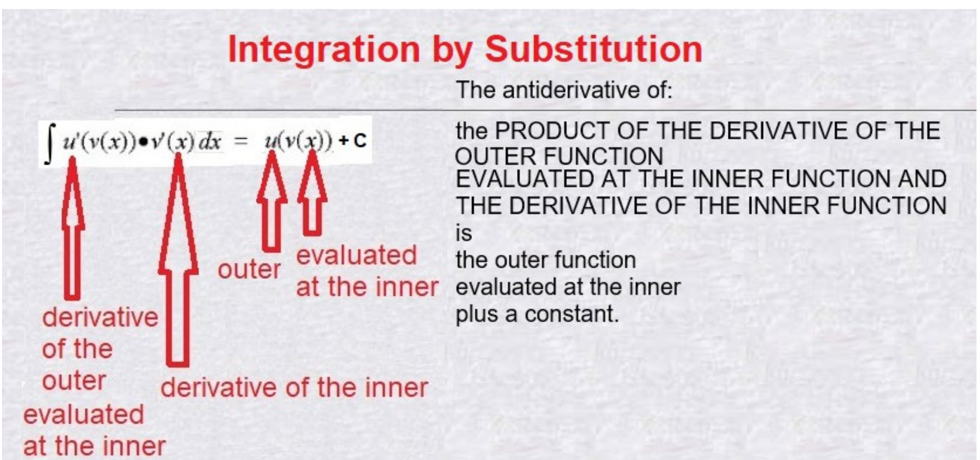
Use Integration by Substitution to Undo:

· **Taking the Derivative of a Composition of Functions,**

· **The Chain Rule for Taking Derivatives.**



Integration by Substitution is a procedure that at first may look too long to complete, but, with a little exposure, much of the time, may be completed mentally.



The integrand must have these as "factors"

- dx, the differential of x
- an "outer function" evaluated at an "inner function,"
- the derivative of the "inner function," and
- may be off by a constant -- need a constant or have an extra constant factor.

The strategy is to:

- rewrite the integral using a new independent variable, u, to replace the more complicated original independent variable, x,
- then complete the easier integration,
- then replace the u with the x if needed.

On this page computation is shown in two areas, the intergal equations and the computation with u.

To Take An Antiderivative By Substitution

$$\int_{\sqrt{\pi}}^{\sqrt{3\pi/2}} \sin(x^2)\, x\, dx =$$

$u = x^2$ new b=u($\sqrt{3\pi/2}$) $= (\sqrt{3\pi/2})^2 = \frac{3\pi}{2}$

$du/dx = 2x$ new a=u($\sqrt{\pi}$) $= (\sqrt{\pi})^2 = \pi$

$du = 2x\ dx$

$\frac{1}{2}du = x\ dx$

$$\frac{1}{2}\int_{\pi}^{3\pi/2} \sin(u)du =$$

$$\frac{1}{2}[-\cos(u)]_{\pi}^{3\pi/2} =$$

$$\frac{1}{2}[\cos(u)]_{3\pi/2}^{\pi} =$$

$$\frac{1}{2}[\cos(\pi) - \cos(\frac{3\pi}{2})] =$$

$$\frac{1}{2}[-1-0] = \frac{-1}{2}$$

1. Declare the "inside function" to be u.
2. Take the derivative of u.
3. Multiply by dx, the differential of x to isolate du, the differential of u.
4. Isolate the dx and any factors containing x on one side of the equation so it is (possible x factors)(dx).
5. If needed, multiply or divide both sides by a constant so (possible x factors)(dx) is alone -- isolated -- on one side.
6. Go back to the original integral.
7. Put u in place of the original function.
8. Remove the (possible x factors)(dx) expression, and replace it with the its equivalent expression which includes du.
9. If the problem is a definite integral, rewrite the bounds of integration in terms of u, not x.
   the new upper bound b = u(b)
   the new lower bound a = u(a)
10. Integrate the rewritten integral.
11. If the problem is an indefinite integral, rewrite the result in terms of x, and add the + C.