Polynomial Functions

$$a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x^1 + a_0x^0$$

Rational Functions

$$\frac{a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x^1 + a_0x^0}{b_nx^n + b_{n-1}x^{n-1} + b_{n-2}x^{n-2} + \ldots + b_2x^2 + b_1x^1 + b_0x^0}$$

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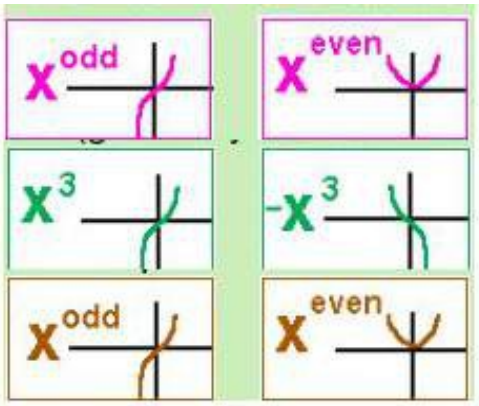
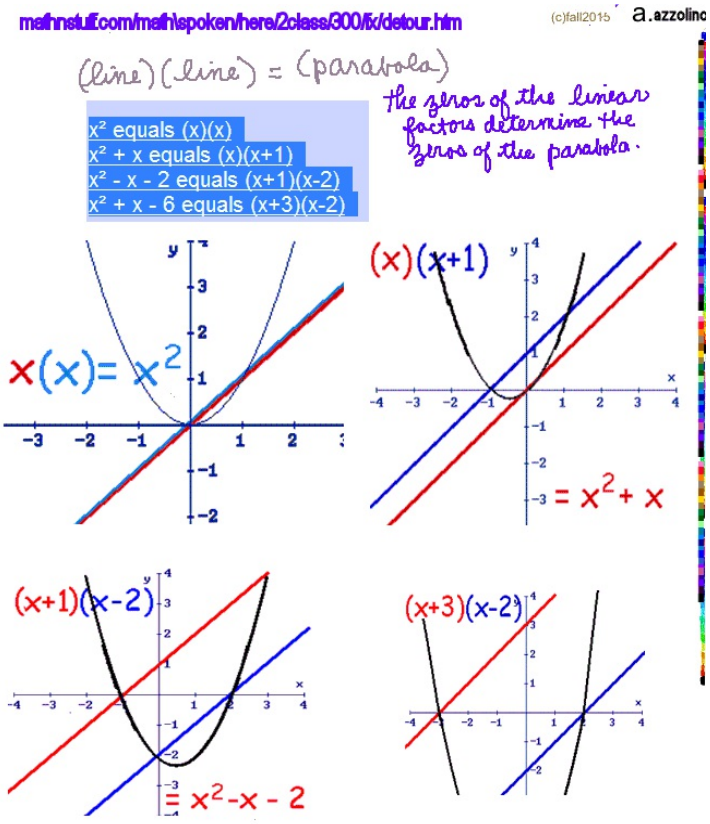
www.mathnstuff.com/math/precalc/pnotes3.htm

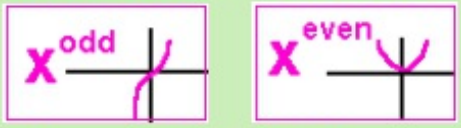


Odd powers start low, end high.
Even powers start and end high
Negative coefficient flips curve over a horizontal line.

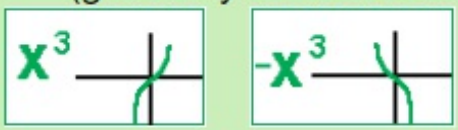


To sketch a polynomial function:

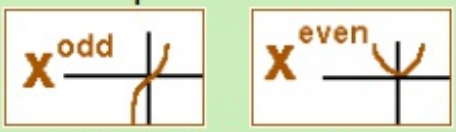
· Note the degree of the polynomial
  -- use it to predict the general shape and end behavior.
  -- even functions "start high & end high"
  -- odd functions "start low & end high"



· Note the coefficient of the term with highest degree
  -- use it to determine if the curve is reflected about the x-axis.
  x^3 "starts low and ends high"
  (generally increases as x increases)
  $-x^3$ "starts high and ends low"
  (generally decreases as x increases)



· Rewrite it by factoring
  identify the linear, quadratic, or other factors.
· Plot the real zeros.
· Note for each root or zero what kind of a root it is
  -- odd powers pass through the x-axis,
  -- even powers touch but do not pass through the x-axis.



· Plot (0, f(0)), the y-intercept.
· Solve: first derivative = 0
  -- to find relative maximums/minimums.
· Determine sign in intervals
  -- using the positiveness or negativeness of each factor.
  -- just "connect the dots."
· Sketch curve.

Rational Zero Theorem --

The rational number, fraction, p/q is a root of the polynomial f(x) if and only if
p is a factor of the constant term and
q is a factor of the leading coefficient.
Examples.

Solve:	Solution(s):	p's, factors of constant term	q's, factors of leading coefficient	p/q's, possible rational root
$3x + 4 = 0$	$x = -4/3$	± 1, ±2, ±4	±1, ±3	± 1/1, ±2/1, ±4/1, ± 1/3, ±2/3, ±4/3
$4x - 3 = 0$	$x = -3/4$	± 1, ±3	±1, ±2, ±4	± 1/1, ±3/1, ± 1/2, ±3/2, ± 1/4, ±3/4
$x + 3 = 0$	$x = -3$	± 1, ±3	±1	± 1/1, ±3/1
$(x + 2)(x - 5) = 0$ $x^2 - 3x - 10 = 0$	$x = -2$, $x = 5$	± 1, ±2, ±5	±1	± 1, ±2, ±5
$x^3 + 5x^2 + 2x - 8 = 0$	$x = ?$	± 1, ±2, ± 4, ±8	±1	± 1, ±2, ± 4, ±8

Rational **(Fraction)** Function Features and How To Find Them

- zeros
  factor the top, write root as (a,0)
- vertical asymptotes
  factor the bottom, write asymptote as x=a
- discontinuties, "holes"
  cancel common factor in top & bottom, find $x \neq a$, find (a, f(a))
- horizontal asymptotes
  complete long division, use remainder
- other asymptotes
  complete long division, use remainder
- end behaviors
  as $x \to +\infty$, $f(x) \to$
  as $x \to -\infty$, $f(x) \to$
  think $f(x) = \frac{Ax^m + \ldots}{Bx^n + \ldots}$

  bigger / smaller: smaller
  smaller / bigger: bigger
  same / same: same
- other features
  point-plot in intervals between the zeros and vertical asymptotes

End Behavior of Rational Functions

is Based on Which is More POWERful.

the top function, $Ax^m + \ldots$, or

the bottom function, $Bx^n + \ldots$

Examine the exponents of the leading terms — $m>n$ $m=n$ $m<n$

Examine the coefficients of the leading terms, $\frac{A}{B}$

$$f(x) = \frac{Ax^m + \ldots}{Bx^n + \ldots}$$