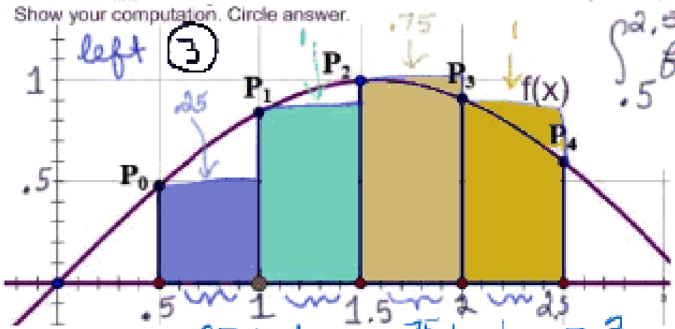
P_0: (0.5, 0.5)
P_1: (1.0, 0.8)

T4m131.pb.set.fa22 F.gif

Show your computation. Circle answer.

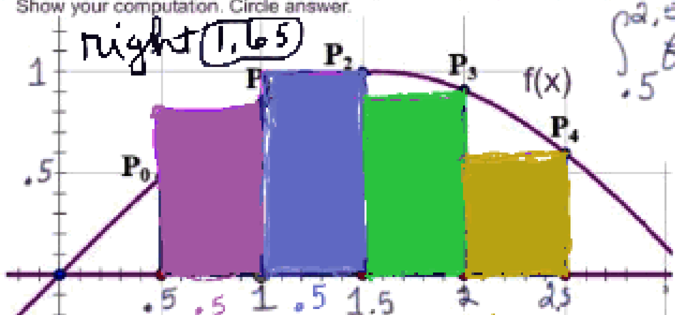


$\int_{.5}^{2.5} f(x)\,dx$

P_0: (0.5, 0.5)
P_1: (1.0, 0.8)
P_2: (1.5, 1.0)
P_3: (2.0, 0.9)
P_4: (2.5, 0.6)

$.25 + 1 + .75 + 1 = 3$

Extra. Approximate using a Reimann boxes/Reimann Sums method as shown in class notes.
Show your computation. Circle answer.



$\int_{.5}^{2.5} f(x)\,dx$

P_0: (0.5, 0.5)
P_1: (1.0, 0.8)
P_2: (1.5, 1.0)
P_3: (2.0, 0.9)
P_4: (2.5, 0.6)

height = .8 is 1 is .9 is .6

$.5(.8) + .5(1) + .5(.9) + .5(.6) = .5(.8+1+.9+.6) =$
$.5(3.3) = 1.65$

16. $\int \cos^3(x)\sin(x)\,dx$

$u = \cos(x)$
$du = -\sin(x)\,dx$
$-du = \sin(x)\,dx$

$\rightarrow -\int u^3\,du = -\frac{u^4}{4} + C \rightarrow -\frac{1}{4}\cos^4(x) + C$

In Exercises 29 – 39, find $f(x)$ described by the given initial value problem.

16. $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx$

$= \frac{x^{1/2}}{1/2} + C = 2\sqrt{x} + C$

26. $\int e^{\pi}\,dx$: e^{π} is a constant

$\int e^{\pi} x^0\,dx = e^{\pi} x + C$

20. $\int 5e^{\theta}\,d\theta = 5e^{\theta} + C$

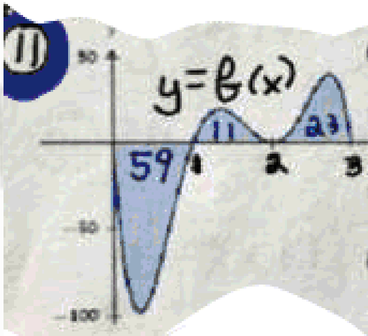
29. $f'(x) = \sin x$ and $f(0) = 2$

$f(x) = \int f'(x)\,dx =$

$\int \sin x\,dx = -\cos(x) + C$

$f(x) = -\cos(x) + C$
$f(0) = 2 = -\cos(0) + C$
$2 = -(1) + C$
$2 = -1 + C$
$3 = C$
$f(x) = -\cos(x) + 3$

11



(a) $\int_0^1 f(x)\,dx$ -59
(b) $\int_0^2 f(x)\,dx$ -48
(c) $\int_0^3 f(x)\,dx$ -26
(d) $\int_1^2 -3f(x)\,dx$ -33

T4m131.pb.set.fa22G.gif

5.4, pg. 246 # 55, 56

5.3, pg. 234 # 6, 26

In Exercises 55 – 58, find $F'(x)$.

55. $F(x) = \int_2^{x^3+x} \frac{1}{t}\, dt = \left[\ln|t|\right]_2^{x^3+x} =$

$\ln|x^3+3| - \ln 2$

56. $F(x) = \int_{x^3}^{0} t^3\, dt$

$= -\int_0^{x^3} t^3\, dt$

$= -\left[\frac{t^4}{4}\right]_0^{x^3}$

$= -\frac{(x^3)^4}{4} = \frac{-x^{12}}{4}$

5.3, pg. 234 # 6, 26

In Exercises 5 – 12, write out each term of the summation and compute the sum.

6. $\sum_{i=-1}^{3} (4i-2) = 4\sum_{i=-1}^{3} i - \sum_{i=-1}^{3} 2 = 4(-1+0+1+2+3) - (2+2+2+2+2)$

$= 4(5) - (10) = 20 - 10 = +10$

or

$(4(-1)-2) + (4(0)-2) + (4(1)-2) + (4(2)-2) + (4(3)-2) =$

$-6 - 2 + 2 + 6 + 10 = 10$

Theorem 5.3.1 states

$$\sum_{i=1}^{n} a_i = \sum_{i=1}^{k} a_i + \sum_{i=k+1}^{n} a_i \text{, so}$$

$$\sum_{i=k+1}^{n} a_i = \sum_{i=1}^{n} a_i - \sum_{i=1}^{k} a_i .$$

Use this fact, along with other parts of Theorem 5.3.1, to evaluate the summations given in Exercises 25 – 28.

26. $\sum_{i=16}^{25} i^3 = \sum_{i=1}^{25} i^3 - \sum_{i=1}^{17} i^3 =$

$\left(\frac{25^4}{4} + \frac{25^3}{2} + \frac{25^2}{4}\right) - \left(\frac{17^4}{4} + \frac{17^3}{2} + \frac{17^2}{4}\right) =$

$105625 - 23{,}409 = 82216$

$$\sum_{i=0}^{n} i^3 = \left(\sum_{i=0}^{n} i\right)^2 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$$

(Nicomachus's theorem)

5.4, pg. 246 # 6, 20, 22, 29, 42, 55, 56

In Exercises 5 – 28, evaluate the definite integral.

6. $\int_0^4 (x-1)^2\,dx =$

$\int_0^4 x^2 dx - \int_0^4 2x\,dx + \int_0^4 1\,dx =$

$\frac{x^3}{3}\Big|_0^4 - \frac{2x^2}{2}\Big|_0^4 + x\Big|_0^4 =$

$\frac{4^3}{3} - 4^2 + 4 =$

$\frac{64}{3} - 16 + 4 = 9\frac{1}{3}$

20. $\int_1^2 \frac{1}{x^3}\,dx = \int_1^2 x^{-3} dx$

$= \frac{x^{-2}}{-2}\Big|_1^2 = -\frac{1}{2}\left[\frac{1}{x^2}\right]_1^2$

$= -\frac{1}{2}\left[\frac{1}{2^2} - \frac{1}{1^2}\right]$

$= -\frac{1}{2}\left[\frac{1}{4} - 1\right] =$

$-\frac{1}{2}\left[-\frac{3}{4}\right] = \frac{3}{8}$

22. $\int_0^1 x^2\,dx =$

$\frac{x^3}{3}\Big|_0^1$

$\frac{1^3}{3} = \frac{1}{3}$

29. Explain why:

(a) $\int_{-1}^{1} x^n\,dx = 0$, when n is a positive, odd integer

(b) $\int_{-1}^{1} x^n\,dx = 2\int_0^1 x^n\,dx$ when n is a positive, even integer.

In Exercises 41 – 46, a velocity function of an object moving along a straight line is given. Find the displacement of the object over the given time interval.

42. $v(t) = -32t + 200$ft/s on $[0, 10]$

42 $v(t) = -32t + 200$ ft/sec on $[0, 10]$

$s(t) = \int(-32t + 200)\,dt$

$s(t) = -16t^2 + 200t + C$

$s(0) = -16(0)^2 + 200(0) + C$

$10 = C$

$\boxed{S(t) = -16t^2 + 200t + 10}$

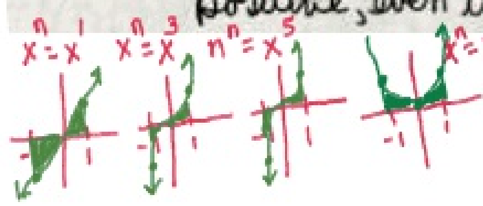
$\int_0^{10} -32t + 200$

$= 400$ ft

29 Explain why:

ⓐ $\int_{-1}^{1} x^n dx = 0$ when n is a positive, odd integer

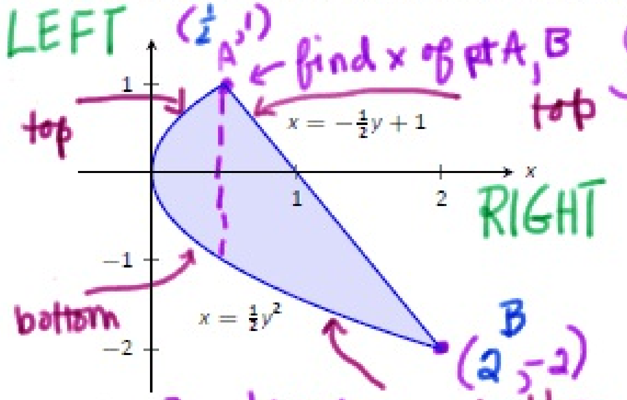
ⓑ $\int_{-1}^{1} x^n dx = 2\int_0^1 x^n dx$ when n is a positive, even integer

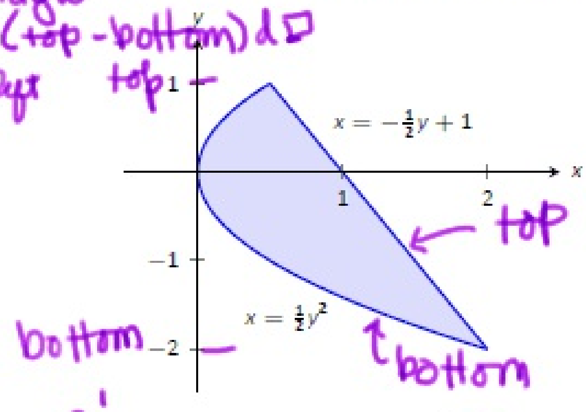


7.1, pg 360 24. In Exercises 21 – 26, find the area of the enclosed region in two ways:

1. by treating the boundaries as functions of x, and
2. by treating the boundaries as functions of y.

$\int_{\text{left}}^{\text{right}} (\text{top} - \text{bottom})\, d\square$





1) $\frac{1}{2}y^2 = -\frac{1}{2}y + 1$

$y^2 = -y + 2$

$y^2 + y - 2 = 0$

$(y-1)(y+2) = 0$

$y = 1 \quad y = -2$

B: $x = \frac{1}{2}y^2$, $x = \frac{1}{2}(-2)^2$, $x = \frac{1}{2}(4) = 2$

A: $x = \frac{1}{2}y^2$, $x = \frac{1}{2}(1)^2$, $x = \frac{1}{2}$

2) Rewrite $x = f(y)$ as $y = f(x)$

LEFT: top $x = \frac{1}{2}y^2$, $2x = y^2$, $\sqrt{2x} = y$, top $y = \sqrt{2x}$

RIGHT: top $x = -\frac{1}{2}y + 1$, $x - 1 = -\frac{1}{2}y$, $-2x + 2 = y$

bottom $y = -\sqrt{2x}$ bottom $y = -\sqrt{2x}$

3rd LEFT

$\int_0^{1/2} (\sqrt{2x} - (-\sqrt{2x}))\, dx = \int_0^{1/2} 2\sqrt{2x}\, dx =$

$2\sqrt{2}\int_0^{1/2} \sqrt{x}\, dx = 2\sqrt{2}\int_0^{1/2} x^{1/2}\, dx = 2\sqrt{2}\left[\frac{2x^{3/2}}{3}\right]_0^{1/2} =$

$\frac{4\sqrt{2}}{3}\left[(\tfrac{1}{2})^{3/2} - (0)^{3/2}\right] = \frac{4\sqrt{2}}{3}\left[\frac{1}{2}\sqrt{\frac{1}{2}}\right] = \frac{2\sqrt{2}}{3\sqrt{2}} = \frac{2}{3}$

$\int_{-2}^{1} \left[(-\tfrac{1}{2}y + 1) - (\tfrac{1}{2}y^2)\right] dy =$

$\int_{-2}^{1} (-\tfrac{1}{2}y^2 - \tfrac{1}{2}y + 1)\, dy =$

$-\frac{1y^3}{2\cdot 3} - \frac{1y^2}{2\cdot 2} + \frac{1y}{1}\Big]_{-2}^{1} =$

$-\frac{1}{6}[y^3]_{-2}^{1} - \frac{1}{4}[y^2]_{-2}^{1} + [y]_{-2}^{1} =$

$-\frac{1}{6}(1+8) - \frac{1}{4}(1-4) + (1+2)$

$-\frac{9}{6} + \frac{3}{4} + 3 = 2\frac{1}{4}$

3rd RIGHT

$\int_{1/2}^{2} \left[(-2x+2) - (-\sqrt{2x})\right] dx =$

$-[x^2]_{.5}^{2} + 2[x]_{.5}^{2} + \frac{2\sqrt{2}}{3}[x^{3/2}]_{.5}^{2} =$

$-[4 - .25] + 2[2 - .5] + \frac{7}{3}$

$-3.75 + 3 + 7/3 =$

$-.75 + 7/3 = 19/12$

$\frac{2\sqrt{2}}{3}\left[x^{3/2}\right]_{1/2}^{2}$

$\frac{2\sqrt{2}}{3}\left[2^{3/2} - (\tfrac{1}{2})^{3/2}\right]$

$\frac{2\sqrt{2}}{3}\left[2\sqrt{2} - \frac{1}{2\sqrt{2}}\right]$

$(2\sqrt{2})(2\sqrt{2})$, $4 \cdot 2$

$\frac{2\sqrt{2}}{3}\left[\frac{8-1}{2\sqrt{2}}\right]$

$\frac{2\sqrt{2}}{3} \cdot \frac{7}{2\sqrt{2}} = \frac{7}{3}$

4) LEFT + RIGHT: $\frac{2}{3} + \frac{19}{12} = 2\frac{1}{4}$