

## Math Tokens

## Tokens Designed for Use in a Digital Age

Tokens
Home
Instructions
5 spread sheets
Theory
Uses
Insert Gif
\& Math History
\& Algebra
\& Algebra
Standards
Spreadsheet
Library
Editing
Notes
\& Pictures
Other
Dictionary
Topics
Algebra
Precalc
Precalc
Functions
Elementary
Middle
HS, HS +
Connect-Dots
Papers
More Stuff

Contact
Open a spreadsheet. Right-click then copy then paste each image on the left into the spread sheet. You've created digital manipulatives, math tokens, things that can move on a spread sheet as one might move tokens on a board.

Your tokens are a set of negative and positive tiles for signed number computation. The spread sheet is required. The images are required. The possibilities are vast in number and may be profound in impact. See instructions and then you might wish to experiment with 5 spread sheets to give you a sample of some of the possibilites. Save a copy of the spreadsheet where you can find it and edit the format to fit your screen.

Use different pictures or tokens for different jobs -- Term Tiles and Tokens for algebra \& prealgebra, Napier's Bones for multiplicity, to multiplying, or to take roots, coins for teaching money, and more specialized spread sheets \& tokens for more specialized tasks like introducing the unit circle, "proving" area formulas, faction /decimal addition / subtraction, finding the resultant vector.

Write a birthday card in hyroglyphics or using Morse Code or nautical flags. Represent and compute as the ancients did in Egypt and Greece and through the Middle Ages using calculi and counting boards. Multiply and divide by Mediation and Duplation - halving and doubling.

Save and edit a math token spreadsheet for yourself (for not-for-profit purposes), but keep the original copy right, author and source page on the table of contents where you also might add notes. Use the resources on the left to assit you in your work.

## Math Tokens Sorted by Math Standards With Notes on Prefered Format \& Use, Proficiencies, Resources

The catagories of "represent," "operate," and "create" might be unfamiliar to the reader.
They are stages of language and math acquisition which inspired math tokens. The most sophisticated stages are "create" and "interpret" and the goal of education is to have each individual have mastery of math at these levels.

Where available, manipulative masters, [m], for concrete work and other resources, [s], are listed after the math token spreadsheet. Spreadsheets open in their own window. Resources generally just link from this page.

Tokens are sorted below by the NJ Math Standards or placed in Other (non-math stuff). Since some are useful in more than one area, they are listed in each area. The Term Tiles \& Tokens -- create.xls, hot.xls, and tiles.xls -- have many uses and are similar.

N: Number Sense and Numerical Operations<br>G: Geometry and Measurement<br>$\underline{\text { P: Patterns, Algebra, Functions }}$<br>D: Data Analysis, Probability, and Discrete Mathematics<br>M: Mathematical Processes -<br>Problem Solving, Communication, Connections, Reasoning, Representations, Technology<br>Q: Other, Non-Math Stuff

N: Number Sense and Numerical Operations

$\underline{\text { fract, }}$ hands $[\underline{s}], \underline{\text { hot }}[\underline{m}, \underline{s}], \underline{\text { apierb }}[\underline{m}, \underline{m}, \underline{s}]$, nomogrf $[\underline{m}, \underline{m}, \underline{m}, \underline{s}]$, nosystm, slide $[\underline{s}]$, strips [ $\mathrm{m}, \mathrm{s}$ ], sumelse [ $\underline{\mathrm{s}}$ ], tiles [ $\mathrm{m}, \mathrm{s}$ ],

## Number Sense and Numerical Operations

9times Nine times tables on your fingers.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both |  |  |  |  | specific |  |  | x | N |

42 Game for 2 Players, uses mental computation and words like multiple, reciprocal, cube, double, prime, ...

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both |  |  |  | x | specific | both | x | x | $\mathrm{N}, \mathrm{M}$ |

100s Hundreds Board

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  | general | both | x | x | N |

abacus Chinese, Japanese, Roman, etc.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| concrete | x | poorly | x |  | general |  |  | N |  |

bases Cubes for the 0, 1st, 2nd, 3rd powers of 2, 3, 4, 5, and 10 and coins ( $0,1 \mathrm{st}$, 2nd, powers of 5 and base 10).

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  | general | both | use spreadsheet |  | N |

board Calculi, single/double boards, horizontal/vertical grooves, Salimus Tablet, Ancient Egypt - Middle Ages

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | $\mathrm{N}, \mathrm{G}, \mathrm{P}, \mathrm{D}, \mathrm{M}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  | general | both | use spreadsheet | N |  |

coins Heads and tails of penny, nickel, dime, quarter, half- and silver dollars, imprinted with cent value or plain.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | $\mathrm{N}, \mathrm{G}, \mathrm{P}, \mathrm{D}, \mathrm{M}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  | general | both | use spreadsheet |  |  |

create For students, though it includes no active hot cells and only some of the "manipulative graphics."

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  | general | both | x | x | $\mathrm{N}, \mathrm{P}, \mathrm{M}$ |

fract Movable fraction bars from $1 / 1$ to $1 / 15$ on multiple sheets.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | $\mathrm{N}, \mathrm{G}, \mathrm{P}, \mathrm{D}, \mathrm{M}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x | x | general | both | use spreadsheet |  | N |

hands Shekels, tokens, in one or two hands, ancient counting board.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| digital | x |  |  |  | specific |  | N |  |  |

hot For students, and includes active hot cells of tiles.xls and only some of the "manipulative graphics."

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | $\mathrm{N}, \mathrm{G}, \mathrm{P}, \mathrm{D}, \mathrm{M}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  | general | both | x | x | $\mathrm{N}, \mathrm{P}, \mathrm{M}$ |

napierb Napier's Bones for multiplication, division, square roots, with instructions.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  |  | both | x | x | N |

nomogrf Nomograph for whole, signed, fraction, decimal computation.

| digital or concrete | represent | operate | create | game | use | teacher/student | masteravailable | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  | general | both | x | x | N, G |

nosystm Hyroglyphics, Acrophonic Greek, Roman, Chinese numerals

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| digital | x |  | x |  | general | both | use spreadsheet |  | N |

slide Slide rules for decimal and fraction addition and subtraction and for log computation.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | $\mathrm{N}, \mathrm{G}, \mathrm{P}, \mathrm{D}, \mathrm{M}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  | general | both | use spreadsheet | x | $\mathrm{N}, \mathrm{G}$ |

strips Multiple strips \& fraction bars.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x | x | general | both | x | x | $\mathrm{N}, \mathrm{G}$ |

sumelse Sum Thing Else Game, keep a match and go again. The most matches wins the game.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both |  |  |  |  | specific | both | x | N | N |

tiles For the parent, teacher, professional educator contains ALL "manipulative graphics" and active hot cells.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  | general | both | x | x | N,P,M |

G: Geometry and Measurement
$\underline{\operatorname{areaf}}[\underline{\mathrm{s}}, \mathrm{m}, \underline{s, m}], \underline{\text { nomogrf }}[\underline{\mathrm{m}}, \underline{\mathrm{m}}, \underline{\mathrm{m}}, \underline{\mathrm{s}}]$, slide $[\underline{\mathrm{s}}], \underline{\text { strips }} \underline{\mathrm{m}}, \underline{\mathrm{s}}]$,

## Geometry and Measurement

areaf Area formulas for a rectangle, parallelogram, triangle, trapezoid, circle.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x |  |  |  | specific |  | some | x | $\mathrm{G}, \mathrm{P}$ |

nomogrf Nomograph for whole, signed, fraction, decimal computation.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | $\mathrm{N}, \mathrm{G}, \mathrm{P}, \mathrm{D}, \mathrm{M}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  | general | both | x | x | $\mathrm{N}, \mathrm{G}$ |

slide Slide rules for decimal and fraction addition and subtraction and for log computation.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | $\mathrm{N}, \mathrm{G}, \mathrm{P}, \mathrm{D}, \mathrm{M}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  | general | both | use spreadsheet | x | $\mathrm{N}, \mathrm{G}$ |

strips Multiple strips \& fraction bars.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | $\mathrm{N}, \mathrm{G}, \mathrm{P}, \mathrm{D}, \mathrm{M}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x | x | general | both | x | x | $\mathrm{N}, \mathrm{G}$ |

P: Patterns, Algebra, Functions
$\underline{\operatorname{areaf}}[\underline{\mathrm{s}}, \mathrm{m}, \underline{\mathrm{s}, \mathrm{m}}]$, create, $[\underline{\mathrm{m}}, \underline{\mathrm{s}}], \underline{\text { hot }}[\underline{\mathrm{m}}, \underline{\mathrm{s}}], \mathrm{jig} 1[\underline{\mathrm{~m}}, \underline{\mathrm{~s}}]$, polrect $[\underline{\mathrm{s}}], \underline{\text { sinelaw }}[\underline{\mathrm{m}}, \underline{\mathrm{s}}]$, tiles $[\underline{\mathrm{m}}, \underline{\mathrm{s}}]$,

## Patterns, Algebra, Functions

areaf Area formulas for a rectangle, parallelogram, triangle, trapezoid, circle.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x |  |  |  | specific |  | some | x | $\mathrm{G}, \mathrm{P}$ |

create For students, though it includes no active hot cells and only some of the "manipulative graphics."

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  | general | both | x | x | $\mathrm{x}, \mathrm{P}, \mathrm{M}$ |

hot For students, and includes active hot cells of tiles.xls and only some of the "manipulative graphics."

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | $\mathrm{N}, \mathrm{G}, \mathrm{P}, \mathrm{D}, \mathrm{M}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  | general | both | x | x | $\mathrm{N}, \mathrm{P}, \mathrm{M}$ |

jig1 Unit circle jig saw puzzle

| digital or concrete | represent | operate | create | game | Use | teacher/student | master available | support material | $\mathrm{N}, \mathrm{G}, \mathrm{P}, \mathrm{D}, \mathrm{M}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both |  |  |  |  | specific |  | x | x |  |

polrect Vector addition with moveable vectors and with spread sheet computations.

sinelaw Sine law-1,2, no solution

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| digital | somewhat |  |  |  | specific |  |  | P |  |

tiles For the parent, teacher, professional educator contains ALL "manipulative graphics" and active hot cells.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | $\mathrm{N}, \mathrm{G}, \mathrm{P}, \mathrm{D}, \mathrm{M}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  | general | both | x | x | $\mathrm{N}, \mathrm{P}, \mathrm{M}$ |

D: Data Analysis, Probability, and Discrete Mathematics deck, dice

## Data Analysis, Probability, and Discrete Mathematics

deck A deck of cards.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M,O |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| concrete | X |  |  | X | general | both |  |  | D |

dice A single die, or pair of dice, sample space and "rollable."

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M,O |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| concrete | X |  |  |  | general | both |  | D |  |

M: Mathematical Processes -
Problem Solving, Communication, Connections, Reasoning, Representations, Technology
$\underline{42}[\underline{\mathrm{~m}}, \underline{\mathrm{~m}}, \underline{\mathrm{~s}}]$, create $[\underline{\mathrm{m}}, \underline{\mathrm{s}}]$, hot $[\underline{\mathrm{m}}, \underline{\mathrm{s}}]$, tiles $[\underline{\mathrm{m}}, \underline{\mathrm{s}}]$,
Mathematical Processes - Problem Solving, Communication, Connections, Reasoning, Representations, Technology
42 Game for 2 Players, uses mental computation and words like multiple, reciprocal, cube, double, prime, ...

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both |  |  |  | x | specific | both | x | x | $\mathrm{N}, \mathrm{M}$ |

create For students, though it includes no active hot cells and only some of the "manipulative graphics."

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  | general | both | x | x |  |

hot For students, and includes active hot cells of tiles.xls and only some of the "manipulative graphics."

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  | general | both | x | x | $\mathrm{N}, \mathrm{P}, \mathrm{M}$ |

tiles For the parent, teacher, professional educator contains ALL "manipulative graphics" and active hot cells.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| both | x | x | x |  | general | both | x | x |  |

O: No Math
hyro, signalf, trans [ $[\underline{s}, \underline{s}]$ [vanish, The Vanishing Math Teacher]

## NO Math

hyro Contains glyphs, cartuches, a column, and pre made phrases with which to create "hyroglyphic" messages.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M,O |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| digital | x |  | x | x | general | both |  | 0 |  |

## signalf Signal flags and Morse Code

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M,O |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| digital | x |  | x |  | specific |  | use spreadsheet |  | 0 |

## trans Contains symbols of phonetic transcriptions and index of sounds.

| digital or concrete | represent | operate | create | game | use | teacher/student | master available | support material | N,G,P,D,M,O |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| digital | x |  | x |  | specific |  |  | x | 0 |

MathTokens.Com © 2010, 2023, A. Azzolino

