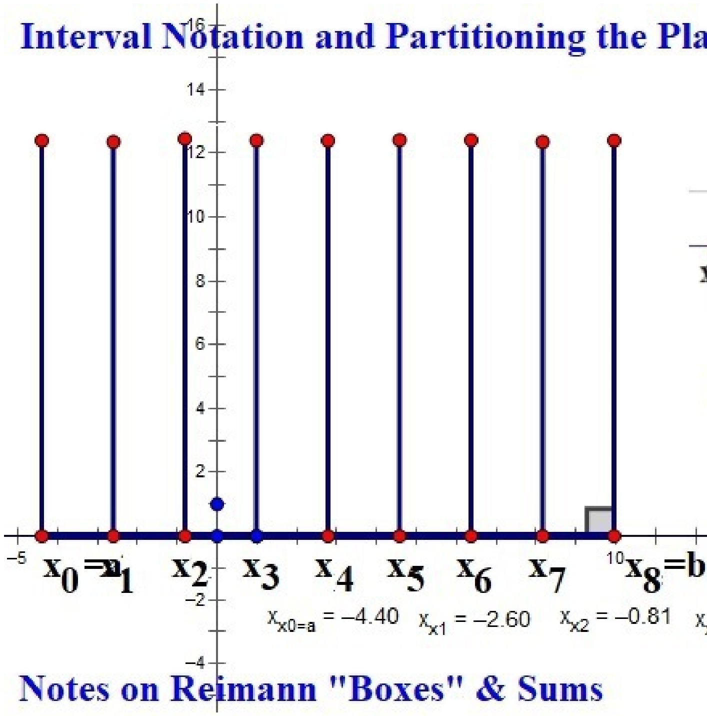
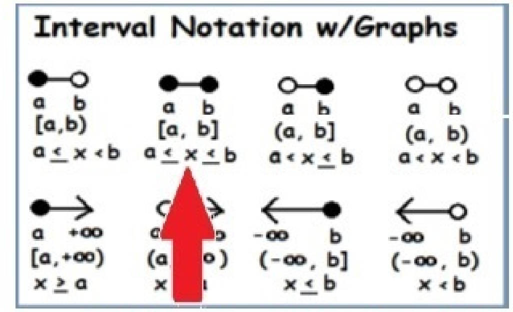
Interval Notation and Partitioning the Plane



Before studying a function graphed on a plane, some of the x-values in a closed interval of the domain are split up equally, partitioned.



[a,b], the closed interval from a to b

Feel free to steal this file and edit as you desire.

In this case, f(x) is defined over the Reals. We are partitioning the domain of that funtion from a to b, [a,b], into 8 equal intervals.

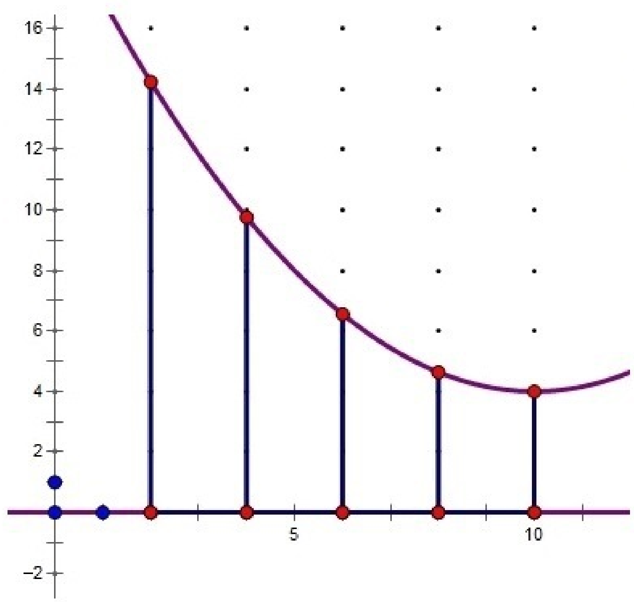
$P = \{[a,x_1], [x_1,x_2], [x_2,x_3], [x_3,x_4], [x_4,x_5], [x_5, x_6], [x_6, x_7], [x_7, b]\}$

Since the interval goes from a to b, its length is b - a, 9.96 - –4.40.
Since this interval is here divided into 8 intervals, each has a width of (b-a)/8 or 1.79. Delta x is 1.79.

Put the pointer on a few of the red points and see how the measures change.

$x_{x0=a} = -4.40$ $x_{x1} = -2.60$ $x_{x2} = -0.81$ $x_{x3} = 0.99$ $x_{x4} = 2.78$ $x_{x5} = 4.58$ $x_{x6} = 6.37$ $x_{x7} = 8.17$ $x_{x8=b} = 9.96$ delta x = 1.79

Notes on Reimann "Boxes" & Sums



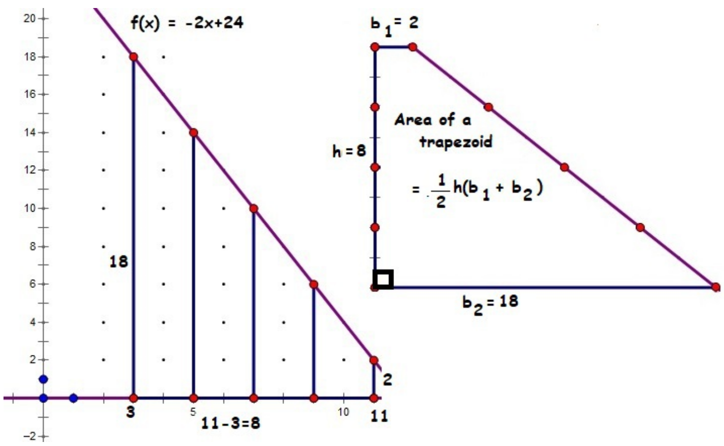
Vocabulary & Symbols

- f(x) -- a function defined over the Reals on the interval [a,b]
- [a,b] -- the interval from a to b including a and b, as in $a \le x \le b$
- n -- the number of sub-intervals in [a,b]
- delta x -- the width of each sub-interval, (b-a)/n
- i -- interval counter, really sub-interval counter
- P -- the partition of the plane divided by the n sub-intervals
  $P = \{ [a=x_0, x_1], [x_1, x_2], [x_2, x_3], \ldots, [x_{n-3}, x_{n-2}], [x_{n-2}, x_{n-1}], [x_{n-1}, x_n=b]\}$
- $(x_i^*, f(x_i^*))$ -- a representative x in an interval & its matching function value, height
  x_i^* might be the
  - LEFT-most x in the interval, or the
  - RIGHT-most x in the interval, or the
  - MIDPOINT of the interval, or
  - some other chosen representative x

On the graph at the left,

1. State: a. f(x) b. [a,b] c. n d. delta x
2. Draw a rectangular box in each interval
   a. with a width of delta x, the width of the entire interval,
   b. with a height of the LEFT-most f(x) for a LEFT Reimann sum,
   with a height of the RIGHT-most f(x) for a RIGHT Reimann sum,
   with a height of the MIDPOINT's f(x) for a MIDPOINT sum,
   with a height of the f(x) of some chosen representative x
3. For each box, compute the area of the box based on the numbers on the graph.
4. Add the areas. This is the Reimann sum.

A Look at the Area Under f(x) = -2x + 24, $3 \le x \le 11$ from Many Points of View



1. Complete the computation for the area of a trapezoid to find the area under f(x) = -2x + 24, $3 \le x \le 11$.

2. Complete the computation of the definite integral to find the area under the curve f(x) = -2x + 24, $3 \le x \le 11$.

$$\int_3^{11} (-2x+24)dx = \left[-x^2+24x\right]_3^{11} =$$

$$[-x^2+24x] - [-x^2+24x] =$$

$$[-(11)^2+24(11)] - [-(3)^2+24(3)] =$$

3. Draw Reimann boxes for the RIGHT Reimann sum and the MIDPOINT Reimann sum. Compute the areas. Compute the sums to approximate the area under f(x) = -2x + 24, $3 \le x \le 11$.